

MAGNETIC PROPERTIES OF MATTER & DIELECTRICS

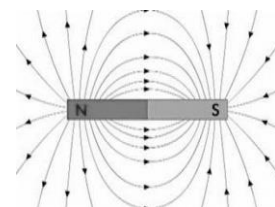
VI Semester B.Sc., Physics Syllabus (Paper 603) Unit – II : Magnetic Properties of Matter

Review of basic formulae : Magnetic intensity, magnetic induction, permeability, magnetic susceptibility, magnetization (M), Classification of Dia, Para, and ferro– magnetic materials. Classical Langevin Theory of dia – and Paramagnetic Domains. Quantum Mechanical Treatment of Paramagnetism. Curie’s law, Weiss’s Theory of Ferromagnetism and Ferromagnetic Domains. Discussion of B-H Curve. Hysteresis and Energy Loss, Hard and Soft magnetic materials

Introduction : Matter is made of atoms which contain charged particles called electrons revolving round the nucleus. These moving electrons produce magnetic field. Also, a current carrying conductor produces magnetic field. All materials are affected in some way or the other by a magnetic field. However, not all materials react the same way.

A **magnetic material** is a material which can attract or repel other substances under the influence of its magnetic field. The action of attracting or repelling a substance depend on the alignment of atomic magnetic moments in the substance in the presence of external magnetic field.

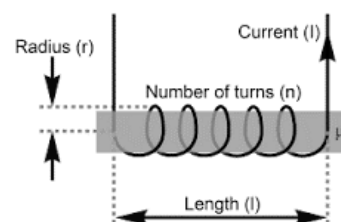
Magnetic Field: It is a region around a magnet where other magnetic materials experience a force. Magnetic field can be described by imaginary lines of force around a magnet which enables other magnetic materials to get repelled or attracted towards it.



Properties of Magnetic field Lines

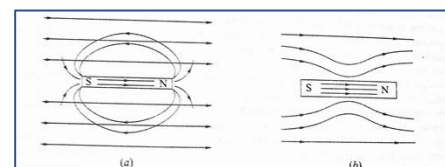
- The Magnetic field lines enter the south pole and comes out of the north pole forming closed loops.
- The magnetic field lines are strong near the poles of the magnet.
- There is no chance of magnetic field lines intersecting each other.
- More the closeness of the magnetic field lines, more is the strength of magnetic field.

Magnetic Field Strength or Magnetic intensity : A current carrying coil (a solenoid) produces a magnetic field H measured as magnetic intensity given by $H = \frac{Ni}{L} \text{ Am}^{-1}$. where N is the number of turns in the solenoid, i is the current and L is the length of the solenoid. H is also called magnetising field.



1. Magnetic permeability

When a magnetic material is placed in a magnetising field H (dia. a), the material acquires magnetism due to induction. The lines of force of the magnetising field concentrate inside the material (dia. b).



Magnetic Flux Density (B) : When a substance is subjected to the magnetic field H, then the number of magnetic field lines that pass through the substance per square meter is known as **Magnetic Flux Density or magnetic induction**.

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The flux density B is directly proportional to magnetic field strength H . Thus $B \propto H$ or $B = \mu H$. The unit of B is **Tesla or weber/m²**.

μ is the constant of proportionality called **absolute Permeability** of the medium and is defined as the ability of a material to allow the magnetic lines of force to pass through it or degree to which a material can be magnetised. It is given by $\mu = \frac{B}{H}$.

If the flux density is established in air or vacuum or a nonmagnetic material, then the above equation is $B_0 = \mu_0 H$. The value of permeability in vacuum is given by $\mu_0 = 4\pi \times 10^{-7} \text{Hm}^{-1}$

The ratio $\frac{\mu}{\mu_0} = \mu_r$ is called the **relative permeability** of the medium given by $\mu_r = \frac{B}{B_0}$. Its value is one for air or a non magnetic material and it is greater than one for a magnetic material.

Magnetic Moment (μ_m) : Motion of electrons in atoms gives rise to the phenomenon called magnetism. Basically, two types of motions are associated with the electrons:

- Motion of electrons in an orbit around the nucleus.
- Spin of electrons around its axis

The spin and orbital moments impart magnetic moment on each electron, forcing them to behave as tiny magnets.

The rotational force (moment) experienced by a magnet when placed in a magnetic field perpendicular to its magnetic axis is defined as the **Magnetic Moment of a Magnet**. For a magnet of length $2l$ and pole strength p , the magnetic moment is $\mu_m = p \times 2l$.

In case of a current loop producing magnetic field $\mu_m = i \times A$, where i is the current and A is the area of the loop. Its unit is Am^2 .

$\mu_m = i \times A = -\frac{e}{T} \times \pi r^2 = -\frac{e\omega}{2\pi} \times \pi r^2 = -\frac{e}{2} \times \omega r^2 = -\frac{e}{2m} \times m\omega r^2 = -\frac{e}{2m} \times L$ Here L is the angular momentum and the negative sign shows that L and magnetic moment are in opposite directions.

2. Magnetisation : Magnetisation is the process of converting a non-magnetic material to a magnetic material.

Intensity of magnetisation (M) is defined as the vector sum of magnetic moments of the atoms (or molecules) contained in unit volume of the material. It is given by

$$M = \frac{\text{Total magnetic moment}}{\text{volume}} = \frac{\sum \mu_m}{V}. \text{ It is a vector and its unit is } \text{Am}^{-1}.$$

Relation between B, M and H : The flux density $B = \mu H = \mu_0 \mu_r H$ or

$$B = \mu_0 \mu_r H + \mu_0 H - \mu_0 H$$

$$\text{or } B = \mu_0 H + \mu_0 H(\mu_r - 1) = \mu_0 H + \mu_0 M \quad \text{where } H(\mu_r - 1) = M.$$

Thus $B = \mu_0(H + M)$ Thus it is seen that the magnetic flux density is equal to the sum of magnetic effect in vacuum and that in the material.

3. Magnetic susceptibility : The intensity of magnetisation for a material is proportional to the magnetising field ($M \propto H$ or $M = \chi H$).

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The **magnetic susceptibility (χ)** of a material is the measure of ease with which the material can be magnetised and is defined as the ratio of intensity of magnetisation induced in the material to the magnetising field. Thus, $\chi = \frac{M}{H}$.

Relation between magnetic susceptibility and relative permeability

The magnetic flux density is $B = \mu_0(H + M)$

or $B = \mu_0(H + \chi H) = \mu_0 H(1 + \chi)$(1) (as $M = \chi H$)

As $B = \mu H$ (2) Comparing (1) and (2) $\mu H = \mu_0 H(1 + \chi)$

Thus $\mu = \mu_0(1 + \chi)$

As $\mu = \mu_0 \mu_r$ we get $\mu_r = (1 + \chi)$.

Classification of magnetic materials Depending on the way the magnetic moments of the electrons in the atoms of a material react to an applied magnetic field, materials can be classified as diamagnetic, paramagnetic and ferromagnetic materials.

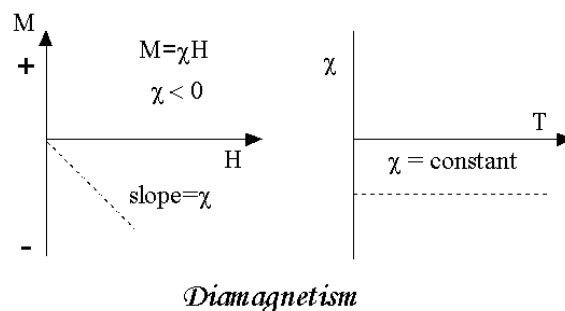
Diamagnetic material : Those substances which are weakly magnetised in a direction opposite to the applied magnetic field are called diamagnetic substances.

Examples: Gold, silver lead, copper, antimony, bismuth, silicon, mercury, water, air, hydrogen, nitrogen etc..

Explanation : In general, the magnetic moments of atoms are oriented such that the vector sum of the magnetic moments of atoms is zero. When such a diamagnetic substance is placed in an external magnetic field, the substance as a whole acquires a net magnetic moment in a direction opposite to the applied field. Atoms of diamagnetic materials do not possess any unpaired electrons.

Characteristics :

1. They possess small and negative susceptibility. ($\chi_m < 0$)
2. The Magnetic moment of the atoms align in the opposite direction to applied magnetic field.
3. They are slightly repelled by a magnetic field.
4. When placed in a non-uniform magnetic field, they tend to move from the stronger to the weaker part of the field.
5. They do not retain the magnetic properties when the external field is removed.
6. Susceptibility is independent of temperature of the material.



Paramagnetic materials : Those substances which are weakly magnetised in the same direction as the applied magnetic field are called paramagnetic substances.

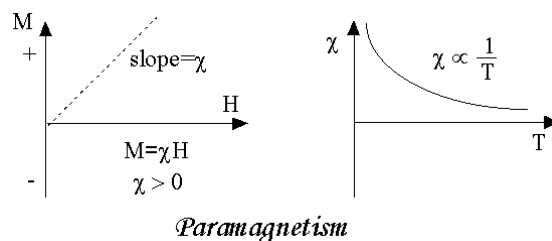
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Examples : Aluminium, platinum, manganese, chromium, sodium, calcium, lithium, tungsten, niobium, oxygen etc..

Explanation : The orbital and the spin magnetic moments of atoms are oriented in such a way that, each atom has a permanent magnetic dipole moment. However, due to thermal motion (vibration), magnetic moments have random orientations. As a result, the net magnetic moment zero. When such a substance is placed in an external magnetic field, each atomic magnet tends to align in the direction of the field. The substance acquires a net magnetic moment and gets magnetised. The atoms of paramagnetic materials possess unpaired electrons.

Characteristics:

1. They possess small and positive susceptibility.
 $\chi_m > 0$ *small*
2. The Magnetic moment of the atoms align in the same direction as that of applied magnetic field.
3. They are slightly attracted by a magnetic field.
4. They tend to move from weaker to the stronger part of the field.
5. Material does not retain the magnetic properties when the external field is removed.
6. Susceptibility depends on the temperature of the material.



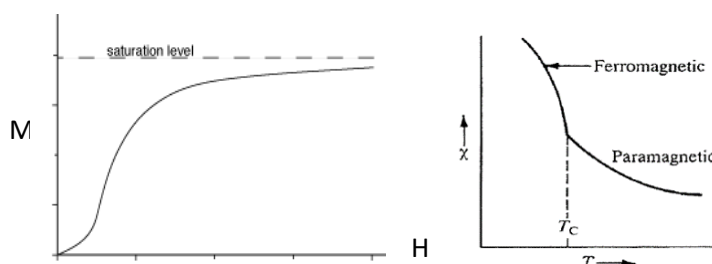
Ferromagnetic materials : Those substances which are strongly magnetised in the same direction as that of applied magnetic field and retains its magnetic moment even after the removal of the applied field are called ferromagnetic materials.

Examples: Iron, cobalt, nickel

Explanation : Ferromagnetism is a special case of paramagnetism. In ferromagnetic substances, the contribution of the spin magnetic moment to the magnetic dipole moment of atoms is very large. They possess strong magnetic properties due to the presence of magnetic domains. In these domains, large number of atomic moments (10^{12} to 10^{15}) are aligned parallel, so that the magnetic force within the domain is strong. When such a material is in the un-magnetized state, the domains are nearly randomly organized and the net magnetic field for the part as a whole is zero. When a magnetizing force is applied, the domains are aligned to produce a strong magnetic field within the part.

Characteristics :

1. They have large and positive susceptibility.
2. They have strong attraction towards magnetic fields.
3. They retain their magnetic properties even after the external field is removed.



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4. They have a strong tendency to move from weaker to the stronger parts of the non uniform external magnetic field.
5. $\vec{B} = \mu_0 (\vec{H} + \vec{M})$ Magnetization is not proportional to the applied field. They exhibit a property called Hysteresis.
6. Susceptibility depends on the temperature of the material.

Differences between Dia, Para and Ferromagnetic substances

Property	Diamagnetism	Paramagnetism	Ferromagnetism
Susceptibility (χ)	$-1 \leq \chi < 0$ (negative and small)	$0 < \chi < \epsilon$ (ϵ a small number) (positive and small)	$\chi \gg 1$ (positive and a large number)
Relative Permeability (μ_r)	$0 \ll \mu_r < 1$ (less than 1)	$1 < \mu_r < 1 + \epsilon$ (slightly greater than 1)	$\mu_r \gg 1$ (much greater than 1)
Retentivity	Zero	Zero or Very low	Very high
Variation of M with H	Linear and small negative slope	Linear and small Positive slope	Non linear variation
Material placed in a uniform magnetic field	Weakly magnetised opposite to the direction of applied magnetic field.	Weakly magnetised in the direction of applied magnetic field	Strongly magnetised in the direction of applied magnetic field
Material placed in a non uniform magnetic field	Tends to move from stronger to weaker regions of the magnetic field	Tends to move from weaker to stronger regions of the magnetic field	Tends to move quickly from weaker to stronger regions of the magnetic field
Effect of temperature	Independent of temperature	$\chi \propto \frac{1}{T}$	$\chi \propto \frac{1}{T - T_c}$ ($T > T_c$)

Classical Langevin's theory of diamagnetism

Diamagnetism is caused by the reaction of the orbiting electron of the atom to an applied magnetic field in accordance with Lenz's law, so that the magnetisation and the susceptibility are negative.

When a diamagnetic substance is placed in an external magnetic field, the induced emf in each atom increases. As a result, the speed of electrons revolving in one direction increases and those revolving in opposite direction decreases. Thus, the substance as a whole acquires a net magnetic moment in a direction opposite to the applied field.

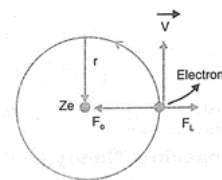
Consider an electron of mass m and charge e rotating about the nucleus of charge Ze in a circular orbit of radius r . Let ω_0 be the angular velocity of the electron.

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The centripetal force acting on the electron is $F_0 = m\omega_0^2 r$ ($\because F = \frac{mv^2}{r}$ and $v = r\omega$) and it is provided by electrostatic force between electrons and nucleus.

$$\text{Thus } m\omega_0^2 r = \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r^2} \quad \text{or} \quad \omega_0^2 = \frac{Ze^2}{4\pi\epsilon_0 m r^3} \quad \dots\dots(1)$$

Let a magnetic field of strength B be applied, (the direction of B is normal and into the plane of the paper). An additional force F_L called Lorentz force acts on the electron given by



$$F_L = -e(v \times B) = -eBr\omega \quad (\because v = r\omega, \quad v \text{ is } \perp \text{ to } B)$$

This leads to change in angular velocity of electron from ω_0 to ω .

$$\text{The condition for stable motion of the electron is given by } m\omega^2 r = \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r^2} - eBr\omega \quad \dots\dots(2)$$

$$\text{Thus } \omega^2 = \frac{Ze^2}{4\pi\epsilon_0 m r^3} - \frac{eB}{m} \omega \quad \text{or} \quad \omega^2 + \frac{eB}{m} \omega - \frac{Ze^2}{4\pi\epsilon_0 m r^3} = 0 \quad \text{or} \quad \omega^2 + \left(\frac{eB}{m}\right)\omega - \omega_0^2 = 0 \quad (\text{from (1)})$$

$$\text{Solving the above quadratic equation in } \omega, \quad \omega = \left\{ -\frac{eB}{m} \pm \sqrt{\left(\frac{eB}{m}\right)^2 + 4\omega_0^2} \right\} \left(\frac{1}{2}\right)$$

$$\omega = -\frac{eB}{2m} \pm \sqrt{\omega_0^2 + \left(\frac{eB}{2m}\right)^2} \quad \text{or} \quad \omega = -\frac{eB}{2m} \pm \omega_0 \quad (\text{since } \frac{eB}{2m} \ll \omega_0 \text{ .second term is neglected})$$

$$\text{Thus } \omega = \pm \omega_0 - \frac{eB}{2m} \quad \dots\dots(3)$$

The change in angular velocity of the electron is $\Delta\omega = \omega - \omega_0 = -\frac{eB}{2m}$.

Under the action of magnetic field, the angular velocity is changed to ω from ω_0 by an amount $\frac{eB}{2m}$. ω is called the Larmor frequency.

$$\text{The magnetic moment of electron is } \mu_m = i \times A = -\frac{e}{T} \times \pi r^2 = -\frac{e\omega}{2\pi} \times \pi r^2 = -\frac{e}{2} \omega r^2 \quad \dots\dots(4)$$

$$\text{The corresponding change in magnetic moment is } \Delta\mu_m = \frac{e}{2} \Delta\omega r^2 = \frac{e}{2} \left(-\frac{eB}{2m}\right) r^2$$

$$\text{or} \quad \Delta\mu_m = -\frac{e^2 r^2 B}{4m} \quad \dots\dots(5)$$

Summing all electrons of the atom (having Z orbital electrons),

$$\text{the total magnetic moment per atom is } \Delta\mu_{m \text{ atom}} = -\frac{Ze^2 B \sum r^2}{4m}$$

If N is the number of atoms per unit volume, then the magnetisation M is given by

$$M = N \Delta\mu_{m \text{ atom}} \quad \text{Thus} \quad M = -\frac{NZe^2 B \sum r^2}{4m} \quad \dots\dots(6)$$

All the electron orbits are not oriented normal to the magnetic field. Hence $\sum r^2$ in equation (6) should be replaced by the average of the square of the projection of orbit radii for various electrons in a plane perpendicular to B.

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Hence $\sum r^2$ is replaced by $\frac{2}{3} \langle r^2 \rangle$.

$$\text{Therefore } M = -\frac{NZe^2B}{4m} \frac{2}{3} \langle r^2 \rangle \quad \text{or} \quad M = -\frac{NZe^2B \langle r^2 \rangle}{6m}$$

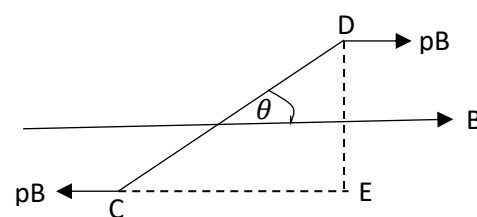
The volume susceptibility of the material is $\chi = \frac{M}{H} = -\frac{NZe^2B \langle r^2 \rangle}{6mH} = -\frac{NZe^2 \mu_0 H \langle r^2 \rangle}{6mH}$
 (since $B = \mu_0 H$) or $\chi = -\frac{\mu_0 e^2}{6m} NZ \langle r^2 \rangle$

The above equation shows that the susceptibility is independent of magnetic field and temperature for a diamagnetic material. It is also seen that χ is directly proportional to the atomic number. The value of χ for most of the diamagnetic substance is of the order of 10^{-6} which is found to be experimentally true.

Langevin's theory of paramagnetism : Curie law

A paramagnetic material possess permanent dipole moments which tend to align along the direction of applied magnetic field which are also disturbed by the thermal agitations. Thus magnetisation depends on the applied field and the thermal energy responsible for agitations.

Every atom of a paramagnetic substances possesses permanent magnetic dipole moment μ_m . The atomic dipoles are considered as tiny magnets (CD) of length $2l$ and pole strength p . When an external magnetic field of strength B is applied, the atomic dipoles experience a force (pB). Let θ be the angle of inclination of the axis of the atomic dipole with the direction of the applied field B .



The torque acting on the dipole is $\tau = pB \times DE = pB \times 2l \sin\theta = \mu_m B \sin\theta \dots (1)$

(since torque = force \times perpendicular distance and $p \times 2l = \mu_m$. Here $CD = 2l$ (diagram))

Now the magnetic potential energy stored in the dipole is $E = \int_{\pi/2}^{\theta} \tau d\theta = \int_{\pi/2}^{\theta} \mu_m B \sin\theta d\theta$

or $E = -\mu_m B \cos\theta \dots (2)$

From classical statistical analysis, the number of atoms or molecules inclined at an angle θ with respect to axis of reference is proportional to $\sin\theta d\theta$. Also on the basis of equipartition of energy, the number of atoms or molecules whose potential energy is E, is proportional to $e^{-E/k_B T}$ where k_B is the Boltzman constant and T is the absolute temperature. Thus if dN is the number of atoms, then

$$dN \propto e^{-E/k_B T} \sin\theta d\theta \quad \text{or} \quad dN = C e^{-E/k_B T} \sin\theta d\theta \dots (3)$$

Substituting for E from (2) in (3) $dN = C e^{\mu_m B \cos\theta / k_B T} \sin\theta d\theta$

Let $\alpha = \frac{\mu_m B}{k_B T}$, then $dN = C e^{\alpha \cos\theta} \sin\theta d\theta \dots (4)$

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To find C : The total number of atomic magnets in unit volume of a paramagnetic substance is $\int dN = C \int_0^\pi e^{\alpha \cos\theta} \sin\theta d\theta$

Let $\cos\theta = x$, then $-\sin\theta d\theta = dx$ Thus $N = -C \int_{+1}^{-1} e^{\alpha x} dx = C \int_{-1}^{+1} e^{\alpha x} dx$

Then $N = C \left[\frac{e^{\alpha x}}{\alpha} \right]_{-1}^{+1} = \frac{C}{\alpha} (e^\alpha - e^{-\alpha})$ or $C = \frac{N\alpha}{(e^\alpha - e^{-\alpha})} \dots (5)$

To find magnetisation M : For an atomic magnet of moment μ_m inclined at an angle θ to the field, the resolved component of its moment in the direction of the field is $\mu_m \cos\theta$. Therefore, the resultant magnetic moment due to all the dN atoms in the direction of the field is $\mu_m \cos\theta dN$.

Hence the total magnetic moment per unit volume called magnetisation is given by $M = \int_0^\pi \mu_m \cos\theta dN$

Substituting for dN from (4) in the above eqn. $M = \int_0^\pi \mu_m \cos\theta C e^{\mu_m B \cos\theta / k_B T} \sin\theta d\theta$

Let $\cos\theta = x$, then $-\sin\theta d\theta = dx$ then $M = C \mu_m \int_{+1}^{-1} x e^{\alpha x} (-dx)$

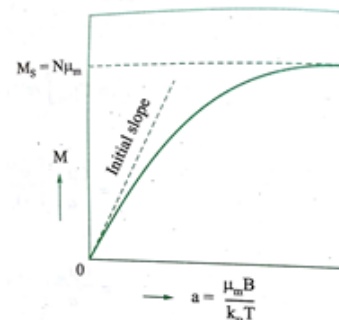
or $M = C \mu_m \int_{-1}^{+1} x e^{\alpha x} dx$, substituting for C from (5) in this equation, integrating and simplifying we get

$$M = N \mu_m \left[\frac{e^{\alpha} + e^{-\alpha}}{e^{\alpha} - e^{-\alpha}} - \frac{1}{\alpha} \right] = N \mu_m \left[\coth\alpha - \frac{1}{\alpha} \right]$$

or $M = N \mu_m L(\alpha) \dots (6)$

where $L(\alpha) = \left[\coth\alpha - \frac{1}{\alpha} \right]$ called the Langevin function

The variation of M with α is as shown.



Case (i) : At low temperature or large applied field, $L(\alpha) \rightarrow 1$. Hence magnetisation in this case is $M = M_s = N \mu_m$. The graph shows that the magnetisation reaches saturation showing that all dipoles align along the direction of the field.

Case (ii) : Under normal conditions α is very small. Then it can be shown that

$$L(\alpha) = \left[\coth\alpha - \frac{1}{\alpha} \right] = \frac{\alpha}{3}. \text{ Therefore } M = N \mu_m \frac{\alpha}{3} = N \mu_m \frac{\mu_m B}{3 k_B T} = \frac{N \mu_m^2 B}{3 k_B T}$$

The susceptibility of the paramagnetic substance is $\chi = \frac{M}{H} = \frac{N \mu_m^2 B}{3 k_B T H}$

or $\chi = \frac{\mu_0 N \mu_m^2}{3 k_B T}$ (since $\frac{B}{H} = \mu_0$) or $\chi = \frac{C}{T}$ where $C = \frac{\mu_0 N \mu_m^2}{3 k_B}$ called Curie constant.

The magnetic susceptibility of a paramagnetic substance varies as $\chi \propto \frac{1}{T}$. This is called

Curie law. It is observed that many paramagnetic materials do not obey the Curie law. They obey the modified law called **Curie-Weiss law** given by $\chi = \frac{C}{T - \theta}$ where θ or also written as T_c which is a constant having dimensions of temperature for a given substance.

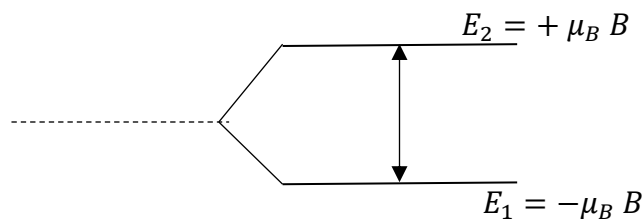
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Failures : 1. The classical theory given by Langevin assumes that the individual atomic or ionic magnetic moments are tiny bar magnets capable of orienting themselves at any arbitrary angle θ with respect to magnetic field direction. But it is observed that only few gases agree with variation of susceptibility with temperature as given by Curie law.

2 This theory could not explain the intimate relation between para and ferro magnetic materials.

Quantum theory of paramagnetism

Quantum theory assumes that the permanent magnetic moments of atoms of paramagnetic materials do not rotate freely, but limited to a finite set of orientations with respect to the magnetic field.



Let N be the number of atoms per unit volume of the material. Considering only the spin moment and no orbital moment, in the presence of the magnetic field H , there are only two energy levels. The spin moment of N_1 atomic dipoles are parallel to the field and N_2 spin moments are antiparallel. The energy acquired by the dipoles aligned parallel to the field is $E_1 = -\mu_B B = -\mu_0 \mu_B H$.

The energy of antiparallel dipoles is $E_2 = +\mu_B B = +\mu_0 \mu_B H$

where B is the magnetic flux density, μ_B is the magnetic moment measured in Bohr magneton (one Bohr magneton = $\frac{eh}{4\pi m}$) of the dipole where a single spin system is assumed and $B = \mu_0 H$.

Thus $E = \pm \mu_B B$. Thus an energy difference exists between spin magnetic dipole moment parallel and anti-parallel to the field. The magnitude of the energy difference is $E_2 - E_1 = \mu_B B - (-\mu_B B) = 2\mu_B B$.

From Maxwell Boltzmann distribution function, $N = N_0 e^{-\frac{E}{k_B T}}$

Thus $N_1 = N_0 e^{-\frac{E_1}{k_B T}} \dots (1)$ and $N_2 = N_0 e^{-\frac{E_2}{k_B T}} \dots (2)$

The total number of dipoles in the system is $N = N_1 + N_2 = N_0 (e^{-\frac{E_1}{k_B T}} + e^{-\frac{E_2}{k_B T}}) \dots (3)$

If a system has only two levels, the equilibrium populations are,

$$\frac{N_1}{N} = \frac{e^{-\frac{E_1}{k_B T}}}{e^{-\frac{E_1}{k_B T}} + e^{-\frac{E_2}{k_B T}}} \dots (4) \quad \text{and} \quad \frac{N_2}{N} = \frac{e^{-\frac{E_2}{k_B T}}}{e^{-\frac{E_1}{k_B T}} + e^{-\frac{E_2}{k_B T}}} \dots (5)$$

As $E_1 = -\mu_B B$ and $E_2 = +\mu_B B$, thus $N_1 = \frac{N e^{\frac{\mu_B B}{k_B T}}}{e^{\frac{\mu_B B}{k_B T}} + e^{-\frac{\mu_B B}{k_B T}}}$ and $N_2 = \frac{N e^{-\frac{\mu_B B}{k_B T}}}{e^{\frac{\mu_B B}{k_B T}} + e^{-\frac{\mu_B B}{k_B T}}}$

Let $\frac{\mu_B B}{k_B T} = x$, then $N_1 = \frac{N e^x}{e^x + e^{-x}}$ and $N_2 = \frac{N e^{-x}}{e^x + e^{-x}}$

The resultant dipole moment per unit volume called **magnetisation** is given by

$$M = \mu_B (N_1 - N_2). \quad \text{Thus } M = N \mu_B \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad \text{or} \quad M = N \mu_B \tanh x \dots (6)$$

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For $x \ll 1$, $\tanh x = x$, and thus $M = N\mu_B x = N\mu_B^2 \frac{B}{k_B T}$ (where $x = \frac{\mu_B B}{k_B T}$)

or $M = N\mu_0 \mu_B^2 \frac{H}{k_B T}$ (7)

Thus susceptibility is $\chi = \frac{M}{H} = \frac{N\mu_0 \mu_B^2}{k_B T}$ or $\chi = \frac{C}{T}$ called Curie law where $C = \frac{N\mu_0 \mu_B^2}{k_B}$.

This theory overcomes drawbacks of the Langevin theory of paramagnetism. But many materials obey a modified law called Curie – Weiss law.

Weiss theory of ferromagnetism

Langevin’s theory of paramagnetism was extended by Weiss to give a theoretical explanation of the behaviour of ferromagnetic materials. He made the following two assumptions:

- (i) Weiss assumed that a ferromagnetic specimen contains a number of small regions (domains) which are spontaneously magnetized. The total spontaneous magnetization is the vector sum of the magnetic moments of the individual domains.
- (ii) The spontaneous magnetization of each domain is due to the existence of an internal molecular field. This tends to produce a parallel alignment of the atomic dipoles. Weiss also assumed that the internal molecular field H_i is proportional to the magnetization M . i.e., $H_i = \gamma M$ where γ is a constant called Weiss constant.

If now an external field H acts on the dipole, then the effective field H_{eff} is given by $H_{eff} = H + H_i = H + \gamma M$... (1)

According to Langevin’s theory of paramagnetism, $M = N\mu_B \tanh x$, where $x = \frac{\mu_B \mu_0 H}{k_B T}$(2)

Thus $M = N\mu_m \tanh\left(\frac{\mu_B \mu_0 H}{k_B T}\right)$ (2)

Replacing H by H_{eff} in the above equation $M = N\mu_m \tanh\left(\frac{\mu_B \mu_0 (H + \gamma M)}{k_B T}\right)$ (3)

Assuming $\mu_m = \mu_B$ called one Bohr magneton as one spin system is considered, the equation becomes $M = N\mu_B \tanh\left(\frac{\mu_B \mu_0 (H + \gamma M)}{k_B T}\right)$(3)

Case (i) : For high temperatures

A ferromagnetic material behaves as a paramagnetic material when temperature is greater than the Curie temperature. For high values of T , the term within the bracket in equation (3) is small and hence $\tanh \theta \approx \theta$. Thus, according to Weiss, for ferromagnetic

materials $M = \frac{N\mu_B^2 \mu_0 (H + \gamma M)}{k_B T}$ (4)

Thus $M = \frac{N\mu_B^2 \mu_0 H}{k_B T} + \frac{N\mu_B^2 \mu_0 \gamma M}{k_B T}$ Rearranging this equation,

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$$M \left(1 - \frac{N\mu_B^2 \gamma \mu_0}{k_B T}\right) = \frac{N\mu_B^2 \mu_0 H}{k_B T} \quad \text{or} \quad M = \frac{\frac{N\mu_B^2 \mu_0 H}{k_B T}}{1 - \frac{N\mu_B^2 \gamma \mu_0}{k_B T}} = \frac{\frac{N\mu_B^2 \mu_0 H}{k_B T}}{\frac{k_B T - N\mu_B^2 \gamma \mu_0}{k_B T}} = \frac{N\mu_B^2 \mu_0 H}{k_B (T - \frac{N\mu_B^2 \gamma \mu_0}{k_B})}$$

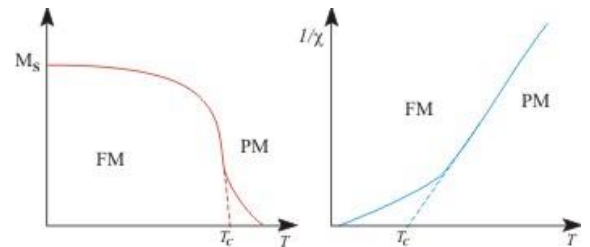
$$\text{Thus } M = \frac{N\mu_B^2 \mu_0 H}{k_B (T - \frac{N\mu_B^2 \gamma \mu_0}{k_B})} \dots \dots (5)$$

$$\text{The susceptibility of a ferromagnetic substance is } \chi = \frac{M}{H} = \frac{N\mu_B^2 \mu_0}{k_B (T - \frac{N\mu_B^2 \gamma \mu_0}{k_B})} \dots \dots (6)$$

$$\text{Thus } \chi = \frac{C}{T - \theta} \dots \dots (7) \quad \text{where } C = \frac{N\mu_B^2 \mu_0}{k_B} \quad \text{and} \quad \theta = \frac{N\mu_B^2 \gamma \mu_0}{k_B} \quad \text{Also} \quad \theta = \gamma C \dots \dots (8)$$

C is called the **Curie constant** and θ also written as T_C is called the Curie temperature. **Curie temperature** is the temperature below which the material shows ferromagnetic behaviour. For values of temperature above θ or T_C , the ferromagnetic substance behaves like a paramagnetic substance. Equation (7) is called the **Curie - Weiss law for ferromagnetics**.

The graph shows the variation of susceptibility and magnetisation with temperature for a ferromagnetic substance. (FM is ferromagnetic and PM is paramagnetic)



The diagram indicates that M_s the saturation magnetisation start dropping at T_C and dies off slowly as there is a divergence in $1/\chi$ at T_C in the paramagnetic region. This is a signature of a phase transition to a spontaneously ordered phase. A positive value of T_C indicates that molecular field is acting in the same direction as of applied field and acts to align the magnetic moments parallel to each other, as should be the case with a ferromagnetic material.

Case (ii) : Ferromagnetic curie temperature

The curie Weiss law $\chi = \frac{C}{T - \theta}$ holds only for temperatures $T > \theta$.

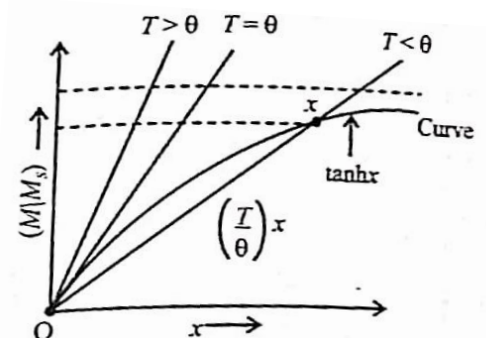
When $T = \theta$ the susceptibility becomes infinite. This shows existence of a non vanishing value of M even though $H = 0$. The existence of magnetisation even in the absence of applied field is called **spontaneous magnetisation**. Thus the spontaneous magnetisation is obtained by putting $H = 0$ in the equation $M = N\mu_B \tanh\left(\frac{\mu_B \mu_0 (H + \gamma M)}{k_B T}\right)$

$$\text{Thus } M = N\mu_B \tanh\left(\frac{\mu_B \mu_0 \gamma M}{k_B T}\right)$$

$$\text{or } M = N\mu_B \tanh x \quad \text{where } x = \frac{\mu_0 \mu_B \gamma M}{k_B T} \dots \dots (9)$$

$$\text{i.e. } \frac{M}{N\mu_B} = \frac{M}{M_{sat}} = \tanh x \dots \dots (10)$$

Here $M_{sat} = N\mu_B$ is the saturation magnetisation due to parallel alignment of all dipoles.



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A graph of $\frac{M}{M_{sat}}$ versus x is as shown. For the temperature $T < \theta$, the value of $\frac{M}{M_{sat}}$ is obtained by the intersection of the line $\left(\frac{T}{\theta}\right)x$ with the $\tanh x$ curve.

For $T \gg \theta$, the spontaneous magnetisation vanishes.

From equation (10) $x = \frac{\mu_0 \mu_B \gamma M}{k_B T}$ or $M = \frac{k_B T x}{\mu_0 \mu_B \gamma}$ (11)

Substituting the value of M from (11) in (10) $\frac{M}{M_{sat}} = \frac{M}{N \mu_B} = \frac{k_B T}{N \mu_0 \mu_B^2 \gamma} x$

From equation (8) $C = \frac{N \mu_0 \mu_B^2}{k}$ and $\theta = \gamma C$ from this equation, $\frac{N \mu_0 \mu_B^2 \gamma}{k} = \theta$

Here $\frac{M}{M_{sat}} = \frac{T}{\theta} x$ (12)

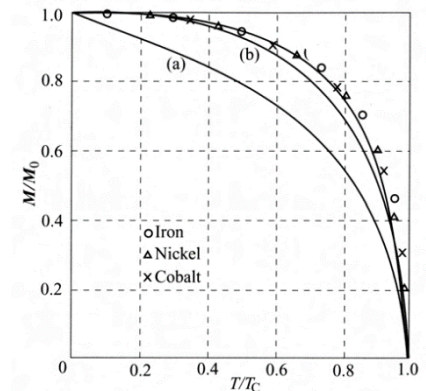
For a given temperature T, the graph of $\frac{M}{M_{sat}}$ versus x represents a straight line with a slope equal to $\frac{T}{\theta}$

From equations (10) and (12), $\frac{M}{M_{sat}} = \tanh x = \frac{T}{\theta} x$

The value of $\frac{M}{M_{sat}}$ for the temperature T is given by the intersection of the straight line and the $\tanh x$ curve, as shown.

For different values T, $\frac{M}{M_{sat}}$ values are calculated. A graph

of this term versus $\frac{T}{\theta}$ is plotted for three materials Fe, Co and Ni. The curve (a) represents the classical function. These curves supports the above analysis for the three metals, even though the curie temperature and saturation magnetisation are different for three metals.



Note : The maximum possible magnetization, or saturation magnetization M_s , of a ferromagnetic material represents the magnetization that results when all the magnetic dipoles in a solid piece are mutually aligned with the external field. There is also a corresponding saturation flux density B_s . The saturation magnetization is equal to the product of the net magnetic moment for each atom and the number of atoms present. For each of iron, cobalt, and nickel, the net magnetic moments per atom are 2.22, 1.72, and 0.60 Bohr magnetons, respectively.

In a ferromagnetic material, coupling interactions cause net spin magnetic moments of adjacent atoms to align with one another, even in the absence of an external field. The mutual spin alignment exists over large volume regions of the crystal called domains.

Ferromagnetic domain theory :

To explain the phenomenon of ferromagnetism, Weiss proposed a hypothetical concept of ferromagnetic domains. Weiss theory of ferromagnetism is also called domain theory of ferromagnetism.

1. According to the domain theory, a ferromagnetic substance consists of a large number of small units (regions) known as Domains.

2. Any ferromagnetic material that is at a temperature below curie temperature T_c is composed of small volume regions in which there is a mutual alignment in the same

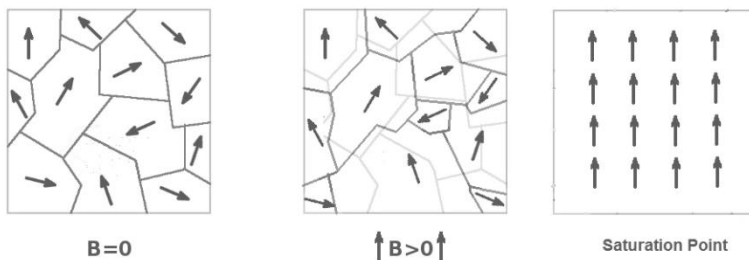
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direction of all magnetic dipole moments (Fig. below). Such a region is called a **domain**, and each one is magnetized to its saturation magnetization.

3. In one domain the magnetic dipole moments of all the atoms are aligned in the same direction. Hence each domain has a resultant magnetic dipole moment. The regions separating magnetic domains are called domain walls, where the magnetization rotates coherently from the direction in one domain to that in the next domain.

4. In the absence of an external magnetic field, various domain vectors have random orientations and hence their resultant magnetic moment is zero.

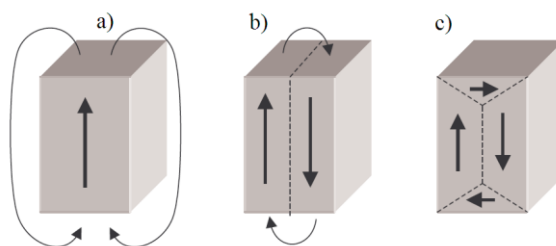
5. When a ferromagnetic substance is kept in the magnetic field, the permanent alignment of domain due to a strong interaction (force) takes place known as exchange coupling. Each domain experience a torque. As a result of this, some domains rapidly rotate and remain aligned parallel to the direction of the field. The magnetisation is said to have taken place.



6. As the strength of the external magnetic field is increased, more and more domains flip and align in the direction of the field. Finally, at a certain stage, practically all domains get aligned in the direction of the field. This is known as **magnetic saturation**. At this stage, a ferromagnetic substance behaves as a permanent magnet and retains its magnetic property (residual magnetism) even if the external magnetic field is removed.

7. Cause for formation of domains : A magnetic material such as iron spontaneously divides into separate domains, rather than exist in a state with magnetization in the same direction throughout the material, is to **minimize its internal energy**.

A ferromagnetic material with a constant magnetization throughout its region will create a large magnetic field extending into the space outside itself (*diagram a*). This requires a lot of *magnetostatic energy* stored in the field.



To reduce this energy, the sample can split into two domains, with the magnetization in opposite directions in each domain (*diagram b*). The magnetic field lines pass in loops in opposite directions through each domain, reducing the field outside the material.

To reduce the field energy further, each of these domains can split further, resulting in smaller parallel domains with magnetization in alternating directions, with smaller amounts of field outside the material.

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The material can further reduce its magnetostatic energy by forming domains with magnetization at right angles to the other domains (*diagram c*). These domains, called flux closure domains, allow the field lines to turn 180° within the material, forming closed loops entirely within the material, reducing the magnetostatic energy to zero.

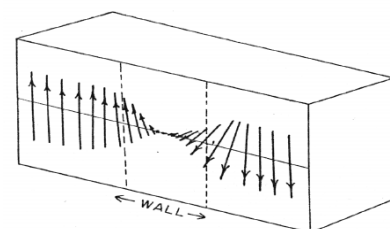
Energies that Determine the Structure of Ferromagnetic Domains : The total magnetic energy of a ferromagnetic material is the sum of the contributions of the following energies: (1) exchange energy, (2) Anisotropy energy, (3) magnetostatic energy, (4) domain wall energy, and (5) magnetostrictive energy.

Exchange energy - The potential energy within a domain of a ferromagnetic solid is minimized when all its atomic dipoles are aligned in one direction (exchange energy).

Anisotropy energy – In ferromagnetic crystals there are easy and hard directions of magnetisation i.e. excess fields are required to magnetise in the hard directions. The excess of energy required to magnetise a specimen in a particular direction over that required to magnetise in the easy direction is called crystalline anisotropy energy.

Magnetostatic energy - Magnetostatic energy is the potential magnetic energy of a ferromagnetic material produced by its external field. This potential energy can be minimized in a ferromagnetic material by domain formation. Since the intensity of the external magnetic field of a ferromagnetic material is directly related to its magnetostatic energy, the formation of multiple domains reduces the magnetostatic energy of a unit volume of material.

Domain wall energy - A domain wall is the boundary between two domains whose overall magnetic moments are at different orientations. Grains changes orientation abruptly and which is about three atoms wide.



However, a domain changes orientation gradually with a domain boundary being ~300 atoms wide. The large width of a domain wall is due to a balance between two forces: exchange energy and magneto crystalline anisotropy. When there is only a small difference in orientation between the dipoles, the exchange forces between the dipoles are minimized and the exchange energy is reduced. Thus, the exchange forces will tend to widen the domain wall.

Thus the net amount that the energy is reduced when a domain splits is equal to the difference between the magnetic field energy saved, and the additional energy required to create the domain wall. The field energy is proportional to the cube of the domain size, while the domain wall energy is proportional to the square of the domain size. So as the domains get smaller, the net energy saved by splitting decreases. The domains keep dividing into smaller domains until the energy cost of creating an additional domain wall is just equal to the field energy saved. Then the domains of this size are stable. In most materials the domains are microscopic in size, around 10^{-4} - 10^{-6} m.

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Magnetostrictive energy - When a ferromagnetic material is magnetized, its dimensions change slightly, and the sample being magnetized either expands or contracts in the direction of magnetization. This magnetically induced reversible elastic strain is called magnetostriction and is of the order of 10^{-6} . The energy due to the mechanical stresses created by magnetostriction is called magnetostrictive energy. The fields of the dipoles may attract or repel each other, leading to the contraction or expansion of the metal during magnetization.

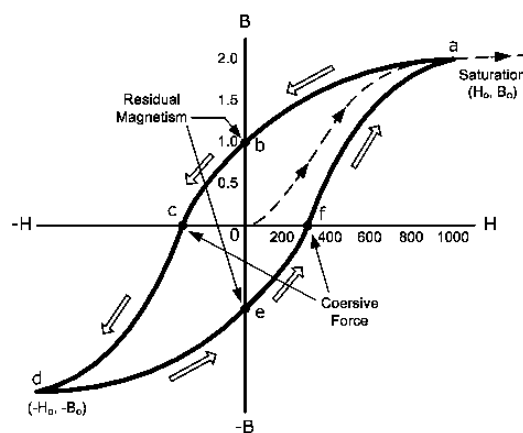
B - H Curve - Magnetic Hysteresis

The B-H curve is the curve showing the variation of magnetic flux density B with the magnetising field H for a material. It is characteristic of the magnetic properties of a material or element or alloy. It talks about how the material responds to an external magnetic field.

Consider a ferromagnetic material in the form a rod with a copper wire wound around it.

A current flowing through the wire produces magnetic field along the axis of the coil given by $H = \frac{Ni}{L}$. This is also called magnetising field. The ferromagnetic material gets magnetised.

The magnetic flux density B which is the sum of magnetising field and the magnetisation acquired by the material is $B = \mu H$ where μ is the absolute permeability of the material. The cycle of magnetisation is as follows :



1. When the core is unmagnetised both B and H will be at zero. (point O on the magnetisation curve)
2. If the magnetising current, i is increased in a positive direction, the magnetic field H increases with i and the flux density B will also increase as shown by the curve from point O to point **a** as it reaches **saturation**. This is a nonlinear variation. i.e. as an H field is applied, the domains change shape and size by the movement of domain boundaries. Initially, the moments of the constituent domains are randomly oriented such that there is no net B (or M) field
3. Now if the current in the coil is reduced to zero, the magnetising field H reduces to zero. But, the flux density B will not reach zero due to the **residual magnetism** present within the core and this is shown by the curve from point **a** to **b**. This is called **Retentivity**. A **hysteresis effect** is produced in which the B field lags behind the applied H field, or decreases at a lower rate. At zero H field, there exists a residual B field that is called remanence, the material remains magnetized in the absence of an external H field.
4. To reduce the flux density at point **b** to zero, the current is reversed in the coil. The magnetising field H which must be applied to nullify the residual flux density is called as "**Coercive field**" or **Coercive force**. This coercive field leads to re-

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- arrangement of molecular magnets until the core becomes unmagnetised at point **c** on the curve.
5. An increase in this reverse current causes the core to be magnetised in the opposite direction and increasing current further will cause the core to reach its saturation point but in the opposite direction, point **d** on the curve.
 6. This point is symmetrical to point **b**. If the current is reduced again to zero the residual magnetism present in the core will be equal to the previous value but in reverse at point **e**.
 7. Again, reversing the current in the coil into a positive direction will cause the flux density to reach zero, point **f** on the curve and as before increasing the current further in a positive direction will cause the core to reach saturation at point **a**.
 8. Then the B-H curve follows the path of **a-b-c-d-e-f-a** as the magnetising current flowing through the coil alternates between a positive and negative value such as the cycle of an AC voltage. This path is called a **Magnetic Hysteresis Loop**.
 9. The magnetization of ferromagnetic substances due to a varying magnetic field lags behind the applied field. This effect is called **hysteresis**, and the term is used to describe any system whose response depends not only on its current state, but also upon its past history.

Hysteresis loss : Magnetic Hysteresis results in the dissipation of energy in the form of heat. The loss of heat is proportional to the area under the magnetic hysteresis loop. It is called Hysteresis loss. It is due to the reversal of magnetization of core whenever it is subjected to alternating magnetizing force. Whenever the core is subjected to an alternating magnetic field, the domain present in the material will change their orientation after every half cycle.

The total magnetic flux ϕ due to current i and flux density B is related as $\phi = A \times dB$ weber where A is the area of cross section of the coil.

For small change in flux, the work done is $dW = \text{ampere turn} \times \text{change in flux} = Ni \times d\phi$.

$dW = NiA dB$ joules where N is the number of turns of the coil.

As $H = \frac{Ni}{L}$ we have $i = \frac{HL}{N}$. Thus $dW = N \left(\frac{HL}{N} \right) A dB = HAL \times dB$ joules

Work done during the complete cycle $W = \int dW = \int HAL \times dB = AL \int H dB$

As $\int H dB$ is the area of the complete hysteresis loop. Thus $W = AL \times \text{area of loop}$

Hence, when the material is taken through a complete cycle, there is loss of energy given by area under the hysteresis curve.

Hard and Soft Magnetic materials : Materials which retain their magnetism and are difficult to demagnetize are called hard magnetic materials.

Ferromagnetic materials in which the magnetic field can be easily reversed are said to be magnetically **soft**. Soft magnetic materials are easy to magnetize and demagnetize.

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S. No.	Hard Magnetic Materials	Soft Magnetic Materials
1	They have very high residual magnetism and high Coercivity.	They have very low residual magnetism and small Coercivity.
2	It retains a strong magnetic field (makes a good permanent magnet), and is difficult to demagnetize	It does not retain a strong magnetic field (does not make a good permanent magnet), and is easy to demagnetize
3	The area enclosed by the B-H curve is large, so it has high hysteresis losses or core losses.	The area enclosed by the B-H curve is small, so it has low hysteresis losses or core losses
4	Susceptibility and permeability are low.	Susceptibility and permeability are high.
5	Magnetic energy stored is high	Magnetic energy stored is low
6	The eddy current loss is high.	The eddy current loss is less because of high resistivity.
7	They are used as permanent magnets and DC magnets.	They are used for transformer and inductance core to minimise energy dissipation. They are also used as electromagnets.
8	Alloys composed of iron, cobalt and aluminium, Chromium steel, Carbon steel etc...	Iron-silicon alloys, nickel-iron alloy and iron, Ferrites etc..

PART A : Descriptive questions

1. (a) Explain the term permeability
(b) What are diamagnetic materials? Explain. Give any four properties of diamagnetic materials.
2. (a) What is magnetic moment? Explain.
(b) What are paramagnetic materials? Explain. Give any four properties of paramagnetic materials.
3. (a) What is magnetisation? Explain.
(b) What are ferromagnetic materials? Explain. Give any four properties of ferromagnetic materials.
4. (a) What is magnetic flux density and susceptibility? Explain.
(b) Arrive at the relation between relative permeability and susceptibility.
- 5 Explain the classical Langevin theory of diamagnetism and hence arrive at the expression for susceptibility.
- 6 Explain the classical Langevin theory of paramagnetism and hence arrive at the expression for susceptibility. Mention its drawbacks.
- 7 Explain the quantum theory of paramagnetism. Arrive at the expression for the susceptibility.
- 8 (a) Explain the Weiss domain theory of ferromagnetism and arrive at the expression for susceptibility.
(b) What are ferromagnetic domains? Explain.
- 9 (a) What is hysteresis? Explain B – H curve.
(b) Distinguish between soft and hard magnetic materials.

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Numerical problems

1 A magnetic material has a magnetisation of 3300 Am^{-1} and flux density of 0.0044 Wbm^2 . Calculate the magnetising field and the relative permeability of the medium. ($\mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}$) Also calculate the susceptibility.

$$[B = \mu_0(H + M) \text{ or } H = \frac{B}{\mu_0} - M = \mathbf{203 \text{ Am}^{-1}}, \quad B = \mu H \mu = \frac{B}{H} = 2.16 \times 10^{-5}, \quad \mu_r = \frac{\mu}{\mu_0} = \mathbf{17.25} \quad \text{As} \\ (1 + \chi) = \mu_r, \quad \chi = \mu_r - 1 = \mathbf{16.25} .]$$

2 A Magnetic field intensity in copper is 10^6 Am^{-1} . If the susceptibility of copper is -0.8×10^{-5} , calculate the magnetic flux density and magnetisation in copper. ($\mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}$)

$$[M = \chi H = -\mathbf{8 \text{ Am}^{-1}} \quad B = \mu_0(H + M) = \mathbf{1.256 \text{ Wbm}^2}]$$

3 The magnetic susceptibility of copper is -0.5×10^{-5} . Calculate the magnetic moment per unit volume in copper when subjected to a magnetic field of magnitude 10^4 Am^{-1} in copper.

$$[M = \chi H = -\mathbf{0.05 \text{ Am}^{-1}}]$$

4 A circular loop of conductor having a diameter of 50 cm carries a current of 100 mA. The loop is placed in a magnetic field having uniform magnetic flux density of 0.05 Wbm^2 with its axis inclined at 60° to the direction of the field. Calculate the values of magnetic dipole moment and torque experienced by the current loop.

$$[\mu_m = i \times A = i \times \pi r^2 = \mathbf{0.0196 \text{ Am}^2}, \quad \tau = \mu_m B \sin\theta = \mathbf{8.48 \times 10^{-4} \text{ Nm}}]$$

5 The magnetic moment of an electron in the ground state of hydrogen atom is one Bohr magneton. Calculate the induced magnetic moment in a field of 1 Wbm^{-2} . (1bohr magneton = $9.27 \times 10^{-24} \text{ JT}^{-1}$) ($e = 1.6 \times 10^{-19} \text{ J}$, $m = 9.1 \times 10^{-31} \text{ kg}$)

$$[\Delta\mu_m = -\frac{e^2 r^2 B}{4m} = -0.197 \times 10^{-28} \text{ Am}^2 \text{ (radius in the first bohr orbit } r = 0.53 \times 10^{-10} \text{ m)} \Delta\mu_m = \\ -\frac{0.197 \times 10^{-28}}{9.27 \times 10^{-24}} = -0.021 \times 10^{-4} \text{ Bohr magneton}]$$

6 The earth has a magnetic dipole moment of $6.4 \times 10^{21} \text{ Am}^2$. What current would have to be set up in a single turn of wire going around the earth at its magnetic equator, if we wish to set up such a dipole? Given the radius of earth = $6.4 \times 10^6 \text{ m}$.

$$[\mu_m = i \times A = i \times \pi r^2 i = \frac{\mu_m}{\pi r^2} = 4.9 \times 10^7 \text{ A}]$$

7 The density and atomic weight of a substance are 7900 kgm^{-3} and 56 respectively. Calculate the spontaneous magnetisation if its magnetic moment is $2.2\mu_B$. Given 1bohr magneton = $9.27 \times 10^{-24} \text{ JT}^{-1}$.

$$[\text{Number of atoms per unit volume} = \frac{\rho N_A}{\text{atomic weight}} = \frac{7900 \times 6.023 \times 10^{26}}{56} = 8.496 \times 10^{28},$$

$$\text{Spontaneous magnetisation } M = N\mu_B = N \times 2.2\mu_B = 1.73 \times 10^6 \text{ Am}^{-1}]$$

8 A magnetic field strength 1800 Am^{-1} produces a magnetic flux of $3 \times 10^{-5} \text{ Wb}$

In an iron bar of cross sectional area 0.2 cm^2 . Calculate the relative permeability. Also find susceptibility.

$$[B = \frac{\phi}{A} = 1.5 \text{ Absolute permeability } \mu = \frac{B}{H} = 8.3 \times 10^{-4} \text{ Hm}^{-1}, \mu_r = \frac{\mu}{\mu_0} = 660.8, \\ \chi = \mu_r - 1 = 659.8]$$

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9 Calculate the susceptibility of copper which is a diamagnet assuming the number of atoms per unit volume as $5 \times 10^{28} m^{-3}$ and the average radius as 0.1 nm. Given charge and mass of an electron as $1.6 \times 10^{-19} C$ and $9.1 \times 10^{-31} kg$ and $\mu_0 = 4\pi \times 10^{-7} Hm^{-1}$.

$$[\chi = -\frac{\mu_0 e^2}{6m} NZ \langle r^2 \rangle = -8.5 \times 10^{-5}]$$

10 Calculate the susceptibility of a paramagnetic salt having atoms per unit volume as $5 \times 10^{28} m^{-3}$ with magnetic flux density of 1T at temperature 300 K. Given $k_B = 1.23 \times 10^{-23} JK^{-1}$ and 1 Bohr magneton = 9.27×10^{-24} .

$$[\chi = \frac{M}{H} = \frac{N\mu_0\mu_B^2}{k_B T} = 1.46 \times 10^{-3}]$$

11 The magnetic susceptibility of silicon is -0.4×10^{-5} . Calculate the flux density and magnetic moment per unit volume when magnetic field of intensity $5 \times 10^5 A/m$ is applied.

$$[B = \mu_0(H + M) = \mu_0 H(1 + \chi) = 0.62 Wb/m^2, M = \chi H = -2.0 A/m.]$$

12 A coil of wire 0.20 m long and having 200 turns carries a current of 10 A. (a) What is the magnitude of the magnetic field strength H? (b) Compute the flux density B if the coil is in a vacuum. (c) Compute the flux density inside a bar of titanium that is positioned within the coil. (d) Compute the magnitude of the magnetization M. ($\chi = 1.81 \times 10^{-4}$)

$$[H = \frac{Ni}{L} = 10^4 Am^{-1}, B = \mu_0 H = 0.0125 Wbm^2, M = \chi H = 1.81 Am^{-1}, \mu_r = 1 + \chi = 1.000181, \mu = \mu_0 \mu_r = 1.256 \times 10^{-6} B = \mu_0 \mu_r H = 1.257 \times 10^{-2} Wbm^2]$$

13 Compute (a) the saturation magnetization and (b) the saturation flux density for cobalt, which has a net magnetic moment per atom of 1.72 Bohr magnetons and a density of $8900 kgm^{-3}$ and atomic weight 58.99. ($\mu_B = 9.27 \times 10^{-24} JK^{-1}$)

$$[N = \frac{\rho N_A}{atomic\ weight} = 9.1 \times 10^{28}, M_S = 1.72 \mu_B N = 1.45 \times 10^6 Am^{-1}, B = \mu_0 M_S = 1.82 T]$$

14 A paramagnetic salt contains 10^{28} atoms/ m^3 , with a magnetic moment of one Bohr magneton. Calculate the paramagnetic susceptibility and the magnetisation produced in a uniform magnetic field of $10^6 Am^{-1}$ at room temperature.

$$[\chi = \frac{M}{H} = \frac{N\mu_0\mu_B^2}{k_B T} = 2.92 \times 10^{-4}, M = \chi H = 292 Am^{-1}]$$

15 Calculate a theoretical value for the saturation magnetization and saturation flux density for pure iron, assuming all magnetic moments due to the four unpaired 3d Fe electrons are aligned in a magnetic field. Given Pure iron has BCC unit cell with lattice constant $a = 0.287$ nm. For BCC each unit has 2 Fe atoms with each iron atom having four bohr magnetons.

$$[M_S = \frac{dipole\ moment}{volume} = \frac{2 \times 4 \times 9.27 \times 10^{-27}}{(0.287^3) \times (10^{-9})^3} = 3.14 \times 10^6 Am^{-1}, B_S = \mu_0 M_S = 3.94 T]$$

16 Find the magnetisation of a bar of length 10 cm and the area of cross section $3 cm^2$. Its magnetic moment is $1 Am^2$. [$V = V = L \times A = 0.3 \times 10^{-4}, M = \frac{\mu_m}{V} = 3.33 \times 10^4 Am^{-1}$]

17 The susceptibility of a material at 400 K is 1.5×10^{-5} . At what temperature the susceptibility will increase to $.8 \times 10^{-5}$?

$$[\chi \propto \frac{1}{T}, \frac{\chi_2}{\chi_1} = \frac{T_1}{T_2} T_2 = 333.33 K]$$

MAGNETIC PROPERTIES OF MATTER & DIELECTRICS

Syllabus : VI Semester : Paper 603 Unit 2 - Dielectrics : Static dielectric constant, polarizability (electronic, ionic and orientation), calculation of Lorentz field (derivation), Clausius-Mosotti equation (derivation), dielectric breakdown, electrostriction (qualitative), electrets. Piezo electric effect, cause, examples and applications.

Dielectrics

A **dielectric** (or **dielectric material**) is an electrical insulator that can be polarized by an applied electric field. When a dielectric is placed in an electric field, electric charges do not flow through the material as they do in an electrical conductor but only slightly shift from their average equilibrium positions causing **dielectric polarization**. Eg: Glass, mica, paraffin, papers, Bakelite etc.

Electric dipole: Two equal and opposite charges +Q and -Q which are separated by a vector distance dx is called an electric dipole.

Electric dipole moment is a measure of the separation of positive and negative electrical charges within a system, that is, a measure of the system's overall polarity. The SI units for electric dipole moment are coulomb-meter (C·m) It is defined as the product of either of the charges and the distance between them.

It is given by $\mu = q \times dx$ and is directed from negative charge to positive charge.

Polar dielectrics	Non – polar dielectrics
If the effective centers of positive and negative charges in the molecules do not coincide with each other (i.e. the effective centres of positive and negative charges are separated by a small distance) even in the absence of any external field, then, such materials are called polar dielectrics.	If the effective centers of the negative charge distribution coincide with the effective center of the positive charges then the effect is to neutralize each other. Such materials are called non – polar dielectrics.
They possess a permanent dipole moment. Eg. H ₂ O, N ₂ O	They do not possess permanent dipole moment. Eg. H ₂ , N ₂ , O ₂ ,.....

Permittivity :

Permittivity is defined as the ratio of electric displacement vector (D) in a dielectric medium to the applied electric field strength (E). Permittivity (ϵ) is a measure of the ability of a material to be polarized by an electric field.

Static dielectric constant ϵ_r or denoted by k is the ratio of the permittivity of the material medium to the permittivity of free space. $\epsilon_r = \frac{\epsilon}{\epsilon_0}$.

1. It is independent of shape or dimensions of the material and it is characteristic of the dielectric medium.
2. Dielectric constant of a material is a macroscopic quantity that measures how effective electric field is, in polarizing the material.

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Expression for static dielectric constant Consider two conducting plates separated by a distance d called a capacitor as shown in the diagram. When the plates are charged, electric field exists between them. Let one flux line originates from each unit charge.

Then q charges having q lines of force received by unit area is called electric displacement denoted by D . It is given by

$$D = \frac{q}{4\pi d^2} \dots\dots(1)$$

As electric field at a point is given by $E =$

$$\frac{q}{4\pi\epsilon_0 d^2} \dots\dots(2) \quad \text{Comparing the two equations we get } E = \frac{D}{\epsilon_0} \text{ or } D = \epsilon_0 E$$

In a dielectric medium $D = \epsilon_0 \epsilon_r E$.

In the absence of dielectric medium, from equation (2) $q = 4\pi d^2 \epsilon_0 E_0$

$$\text{or } q = \epsilon_0 E_0 A \dots\dots(3)$$

The capacitance in the absence of dielectric medium is $C_0 = \frac{q}{V_0}$ where V_0 is the potential difference between the plates given by $V_0 = E_0 d$ (since $E = -\frac{dV}{dx}$).

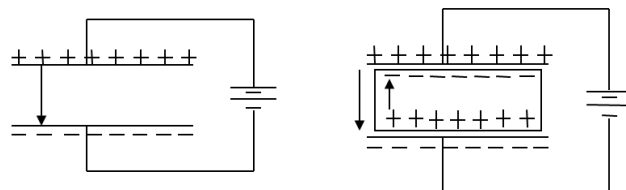
$$\text{Thus } C_0 = \frac{\epsilon_0 A}{d} \dots\dots(4) \text{ (as } \epsilon_r = 1 \text{ for air or vacuum).}$$

When a dielectric medium is introduced between the plates, the capacitance is $C = \frac{q'}{V}$

$$\text{or } C = \frac{\epsilon_0 \epsilon_r E A}{V} \quad \text{where } V = E d \quad \text{Thus } C = \frac{\epsilon_0 \epsilon_r A}{d} \dots\dots(5)$$

$$\text{Dividing (5) by (4) we get } \frac{C}{C_0} = \epsilon_r \quad \text{or } \epsilon_r = \frac{C_{\text{material}}}{C_{\text{vacuum}}}.$$

By introducing dielectric we see that $V < V_0$, $E < E_0$. Thus $C > C_0$



Note : The static dielectric constant is the factor by which the capacitance of a capacitor is increased when vacuum is substituted by a dielectric medium which fills the entire region where electric field would be set up on subjecting the capacitor to a static electric potential.

Dielectric polarization (Electrical polarization) :

The displacement of charges in the molecules of a dielectric under the action of an applied electric field, leading to development of dipole moment is called **dielectric polarization**.

Because of dielectric polarization, positive charges are displaced in the direction of the field and negative charges shift in the opposite direction. This creates an internal electric field that reduces the overall field within the dielectric itself.

From the macroscopic point of view, sum of the dipole moments in a given volume is given by $\sum_{i=1}^{N\Delta V} \mu_i = N \bar{\mu} = P \Delta V$ where N is the number of molecules per unit volume, $\bar{\mu}$ is the average dipole moment and P is the dipole moment per unit volume called electric polarization.

Expression for dielectric polarisation : In the absence of dielectric, the electric field E_0 between the plates of a capacitor is

$$E_0 = \frac{q}{A \epsilon_0} \dots\dots(1) \quad \text{(since } q = \epsilon_0 E_0 A \text{)}$$

When a dielectric medium is introduced between the plates, due to electric polarisation, surface charges q' are developed on the dielectric producing electric field opposite to applied field.

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Thus net electric field decreases. i.e. $E = \frac{q}{A \epsilon_0 \epsilon_r} \dots\dots(2)$

Also, in the presence of dielectric medium, $E = \frac{q-q'}{A \epsilon_0} = \frac{q}{A \epsilon_0} - \frac{q'}{A \epsilon_0} \dots\dots(3)$

Comparing (2) and (3) $\frac{q}{A \epsilon_0 \epsilon_r} = \frac{q}{A \epsilon_0} - \frac{q'}{A \epsilon_0}$ or $\frac{q}{A \epsilon_r} = \frac{q}{A} - \frac{q'}{A}$

Thus $\frac{q}{A} = \frac{q \epsilon_0}{A \epsilon_r \epsilon_0} + \frac{q'}{A}$ or $D = \epsilon_0 E + P$ where $D = \frac{q}{A}$ and $P = \frac{q'}{A}$ called electric polarisation.

or $P = D - \epsilon_0 E$ or $P = \epsilon E - \epsilon_0 E = \epsilon_r \epsilon_0 E - \epsilon_0 E$

Thus $P = \epsilon_0 E (\epsilon_r - 1)$ or $\epsilon_r - 1 = \frac{P}{\epsilon_0 E} = \chi$. This is called **electric susceptibility** of the dielectric medium. The **electric susceptibility** χ of a dielectric material is a measure of how easily it polarizes in response to an electric field. This, in turn, determines the electric permittivity of the material and thus influences many other phenomena in that medium, from the capacitance of capacitors to the speed of light.

The electric dipole moment $\mu \propto E$ or $\mu = \alpha E$, where α is the **dielectric polarisability of the medium**.

As, $P = \epsilon_0 (\epsilon_r - 1) E = \epsilon_0 \chi E$ we see that P is directly proportional to E. i.e. $P \propto E$.

Also $P = N \mu = N \alpha E$, where N is the number of atoms per unit volume.

Thus $\epsilon_0 (\epsilon_r - 1) E = N \alpha E$ or $\alpha = \frac{\epsilon_0 (\epsilon_r - 1)}{N}$.

Types of Electric polarization

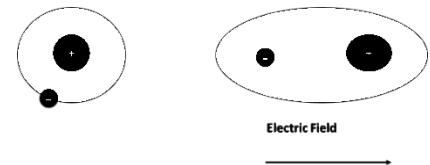
There are four different mechanisms through which electrical polarization can occur in dielectric materials when they are subjected to an external electric field. They are

1. Electronic polarization,
2. Ionic polarization
3. Orientation polarization
4. Space charge polarization

1. Electronic polarization

The electronic polarization occurs due to the displacement of the positive and negative charges in a dielectric material due to the application of an external electric field as shown.

The development of dipole moment occurs throughout the material, so whole material will be polarized.



Expression for electronic polarisability

Let Ze represent nuclear charge surrounded by negative charge as shown in the diagram.

The negative charge density is given by $\rho = -\frac{Ze}{\frac{4}{3}\pi R^3} = -\frac{3Ze}{4\pi R^3} \dots\dots(1)$ since $\rho = \frac{\text{charge}}{\text{volume}}$

where R is the radius of the atom.

When the system is subjected to electric field E, the charges experience electric force given by $F_Z = -ZeE. \dots\dots(2)$

It is assumed that only electron cloud is displaced by the field. Let this displacement be x. Then the Coulomb force is $F_C = Ze \times \frac{\text{charge enclosed in the sphere of radius } x}{4\pi \epsilon_0 x^2} \dots\dots(3)$

The charge enclosed by the sphere is $q = \frac{4}{3} \pi x^3 \rho \dots\dots(4)$

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Substituting for ρ from (1) in (4), we get $q = \frac{4}{3} \pi x^3 \times \left(-\frac{3Ze}{4\pi R^3}\right) = -\frac{Ze x^3}{R^3}$ (5)

Hence the coulomb force is given by putting (5) in (3) for the charge, Hence

$$F_C = \frac{Ze}{4\pi\epsilon_0 x^2} \times -\frac{Ze x^3}{R^3} = -\frac{Z^2 e^2 x}{4\pi\epsilon_0 R^3} \quad \dots\dots(6)$$

In the equilibrium position the two forces are equal i.e. $F_Z = F_C$.

Comparing equations (2) and (6) we get $-ZeE = -\frac{Z^2 e^2 x}{4\pi\epsilon_0 R^3}$

or $x = \frac{4\pi\epsilon_0 R^3 E}{Ze}$. Thus the displacement of the electron cloud is proportional to the applied field. The dipole moment due to electronic polarisation is given by

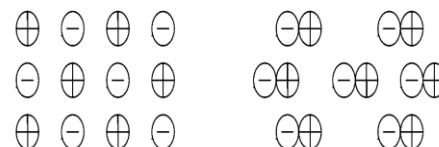
$$\mu_e = Ze x = Ze \frac{4\pi\epsilon_0 R^3 E}{Ze} = 4\pi\epsilon_0 R^3 E$$

Thus $\mu_e = 4\pi\epsilon_0 R^3 E$ or $\mu_e \propto E$ or $\mu_e = \alpha_e E$ where $\alpha_e = 4\pi\epsilon_0 R^3$ α_e is called the electronic polarisability.

Note : Electronic polarizability is proportional to the volume of the atoms and is independent of temperature. It occurs only at optical frequencies of 10^{15} Hz in the optical region. Also electronic polarisation is $P_e = N\alpha_e E$

2. Ionic polarisation

This occurs only in materials which are ionic. An applied field acts to displace positive charges in one direction (cation) which gives rise to a net dipole moment.

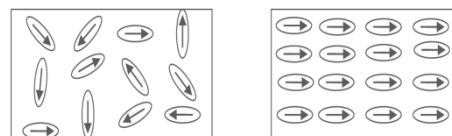


When an Electric field is applied to the molecule, the positive ions are displaced by X_1 to the negative side of electric field and negative ions are displaced by X_2 to the positive side of field. The resultant dipole moment $\mu = q (X_1 + X_2)$. This polarization occurs at frequency 10^{13} Hz (IR). It is a slower process compared to electronic polarization. It is independent of temperature. The ionic polarisation is given by $P_i = N\alpha_i E$ where α_i is the **ionic polarisability**.

This type of polarisation typically occurs in ionic crystal elements such as NaCl and KCl. This mechanism contributes to relative permittivity. Inside these materials, there is no net polarisation in the absence of electric field. This is because dipole moments of the negative ions are cancelled out by the positive ions. When an external field is applied, the ions are displaced, this leads to an induced polarization

3. Orientational polarisation

This is found only in substances that possess permanent dipole moments. Polarization results from a rotation of the permanent dipoles in the direction of the applied field. It is strongly temperature dependent and decreases with increase of temperature.



It is also called dipolar or molecular polarization. The molecules such as H_2 , N_2 , O_2 , Cl_2 , CH_4 , CCl_4 etc., does not carry any dipole because centre of positive charge and centre of negative charge coincides. On the other hand molecules like CH_3Cl , H_2O , HCl , ethyl acetate (polar molecules) carries dipoles even in the absence of electric field.

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In the case of a CH_3Cl molecule, the positive and negative charges do not coincide. The Cl^- has more electro negativity than hydrogen. Therefore, the chlorine atoms pull the bonded electrons towards them more strongly than hydrogen atoms. Therefore, even in the absence of field, there exists a net dipole moment. In the presence of the field there is alignment of dipoles leading to polarization.

It occurs at a frequency 10^6 Hz to 10^{10} Hz. It is slow process compared to ionic polarization. It greatly depends on temperature. The orientational polarisation is $P_o = N\alpha_o E$ where α_o is the **orientational polarisability** given by $\alpha_o = \frac{\mu^2}{3kT}$.

4. Space charge polarisation

The space-charge polarization occurs due to the diffusion of ions, along the field direction, thereby giving rise to redistribution of charges in the dielectrics.



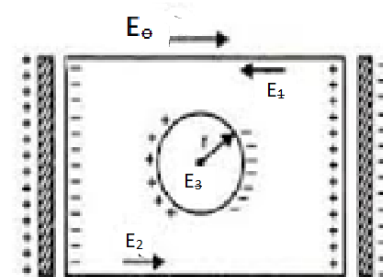
Space charge polarization occurs in multiphase dielectric materials in which there is a change of resistivity between different phases. The space charge polarization is not an important factor in most common dielectrics.

Interfacial or space charge polarization occurs when there is an accumulation of charge at an interface between two materials or between two regions within a material because of an external field. This can occur when there is a compound dielectric, or when there are two electrodes connected to a dielectric material. This polarization is usually observed in amorphous or polycrystalline solids. The electric field will cause a charge imbalance. Mobile charges in the dielectric will migrate over to maintain charge neutrality. This then causes interfacial polarization.

Total polarisation is given by $P = P_e + P_i + P_o$.

Expression for the internal or the local field (Lorentz field)

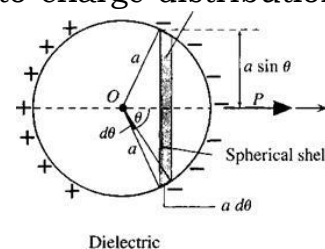
The electric field experienced by an atom in the dielectric medium is the local electric field. To evaluate the field, a dipole inside the dielectric medium in an applied external electric field is considered. An imaginary hollow cavity called the Lorentz cavity is considered around this dipole whose radius is greater than radius of the atom. The internal field at the atom is given by $E_{local} = E_0 + E_1 + E_2 + E_3 \dots(1)$



where E_0 is the applied external electric field, E_1 is the depolarising field i.e. field produced due to surface charges on the dielectric, E_2 is the electric field due to charges on the surface of Lorentz cavity and E_3 is the electric field due to charge distribution inside the cavity which is dependent on structure of the material.

(a) To find E_0 : The electric displacement D due to the applied field is given by $D = \epsilon_0 E + P$

where $D = \frac{q}{A}$ and $P = \frac{q'}{A}$ called electric polarisation.



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Also $D = \epsilon_0 E_0$. E in the above equation is the average electric field due to both applied field and field due to polarisation.

The above equation becomes $\epsilon_0 E_0 = \epsilon_0 E + P$ or $E_0 = E + \frac{P}{\epsilon_0}$ (2)

(b) To find E_1 : The application of field E_0 will polarise the medium producing another field which acts in the opposite direction called the depolarising field. Considering only this field, equation (2) can be written as $E_1 = -\frac{P}{\epsilon_0}$ (3) where E is taken as zero. (In general for an infinite slab of dielectric medium, this depolarising field is $E_1 = -\frac{NP}{\epsilon_0}$, where N is the depolarising factor.

(c) To find E_2 : This field is due to surface charges on the Lorentz cavity or Lorentz sphere. If dS is the area of a ring on the sphere of radius a lying between θ and $d\theta$, then its surface area $dS = 2\pi x dx$.

From the diagram, $\sin\theta = \frac{x}{a}$ or $x = a \sin\theta$ and $d\theta = \frac{dx}{a}$ or $dx = a d\theta$

Hence $dS = 2\pi x dx = 2\pi a^2 \sin\theta d\theta$

If dq is the charge on this area, then it is given by the product of normal component of polarisation and the surface area i.e. $dq = P \cos\theta dS$ or $dq = P 2\pi a^2 \sin\theta \cos\theta d\theta$

The electric field at the centre of the Lorentz cavity due to this ring is given by

$$dE_2 = \frac{1}{4\pi a^2} dq \cos\theta$$

Substituting for dq in this equation we get $dE_2 = \frac{1}{4\pi\epsilon_0 a^2} P 2\pi a^2 \sin\theta \cos\theta d\theta \cos\theta$

$$\text{or } dE_2 = \frac{P}{2\epsilon_0} \cos^2\theta \sin\theta d\theta .$$

The total electric field is $E_2 = \frac{P}{2\epsilon_0} \int_0^\pi \cos^2\theta \sin\theta d\theta$.

Let $\cos\theta = z$, then $dz = -\sin\theta d\theta$ Also $\cos 0 = +1$ and $\cos \pi = -1$.

Thus the above equation is $E_2 = -\frac{P}{2\epsilon_0} \int_{+1}^{-1} z^2 dz = -\frac{P}{2\epsilon_0} \left[\frac{z^3}{3} \right]_{+1}^{-1} = -\frac{P}{6\epsilon_0} (-1 - 1) = \frac{P}{3\epsilon_0}$.

Thus the electric field is $E_2 = \frac{P}{3\epsilon_0}$ (4)

(d) To find E_3 : This field is due to electric dipoles present within the cavity which depends on crystal structure.

For cubic structure this field is zero, i.e. $E_3 = 0$ (5)

The total electric field at the site of the atom inside the dielectric medium is obtained by adding all these fields. Substituting for the field expressions from (2), (3), (4) and (5) in (1) i.e. $E_{local} = E_0 + E_1 + E_2 + E_3$ we get

$$E_{local} = E + \frac{P}{\epsilon_0} + \left(-\frac{P}{\epsilon_0} \right) + \frac{P}{3\epsilon_0} + 0 = E + \frac{P}{3\epsilon_0}$$

Thus the local or the internal electric field is $E_{local} = E + \frac{P}{3\epsilon_0}$.

Derivation of Clausius Mossotti equation Consider a dielectric medium which has predominantly the electronic polarisation and other effects are negligible. The dipole moment and the electronic polarisation are given by

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$\mu_e = \alpha_e E$, where α_e is the electronic polarisability of the medium and

$P_e = N\mu_e = N\alpha_e E_l$ (1) where E_l is the local electric field and where N is the number of atoms per unit volume.

The electric displacement is $D = \epsilon_0 E + P$ or $P = D - \epsilon_0 E$

Dividing the above equation by E, we get $\frac{P}{E} = \frac{D}{E} - \epsilon_0$

As $D = \epsilon E = \epsilon_r \epsilon_0 E$, or $\frac{D}{E} = \epsilon_r \epsilon_0$,

Thus the above equation becomes $\frac{P}{E} = \epsilon_r \epsilon_0 - \epsilon_0 = \epsilon_0 (\epsilon_r - 1)$

or $P = \epsilon_0 (\epsilon_r - 1) E$ (2)

The local or the internal field is $E_l = E + \frac{P}{3\epsilon_0}$ (3)

Substituting for E_l from (3) in (1) $P = N\alpha_e \left(E + \frac{P}{3\epsilon_0} \right)$ (4)

Comparing (2) and (4) $N\alpha_e \left(E + \frac{P}{3\epsilon_0} \right) = \epsilon_0 (\epsilon_r - 1) E$

$N\alpha_e E + \frac{N\alpha_e P}{3\epsilon_0} = \epsilon_0 (\epsilon_r - 1) E$

Substituting for P in the above equation from (2) $N\alpha_e E + \frac{N\alpha_e \epsilon_0 (\epsilon_r - 1) E}{3\epsilon_0} = \epsilon_0 (\epsilon_r - 1) E$

$N\alpha_e \left(1 + \frac{\epsilon_0 (\epsilon_r - 1)}{3\epsilon_0} \right) = \epsilon_0 (\epsilon_r - 1)$

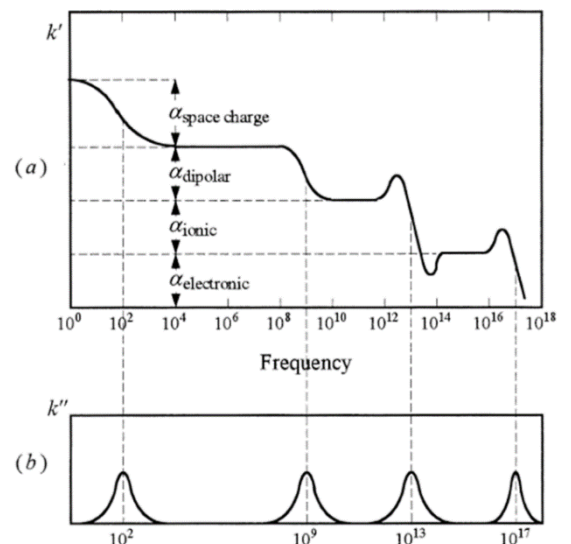
$\frac{N\alpha_e}{3\epsilon_0} (3\epsilon_0 + \epsilon_0 (\epsilon_r - 1)) = \epsilon_0 (\epsilon_r - 1)$ or $\frac{N\alpha_e}{3\epsilon_0} (\epsilon_r + 2)\epsilon_0 = \epsilon_0 (\epsilon_r - 1)$

or $\frac{(\epsilon_r - 1)}{(\epsilon_r + 2)} = \frac{N\alpha_e}{3\epsilon_0}$. This is called **Clausius Mossotti equation** and it holds good for crystals of high degree of symmetry. This equation relates the dielectric constant ϵ_r which is the macroscopic property of the dielectric medium to α_e i.e. electronic polarisability which is the atomic property of the material.

Frequency dependence of dielectric constant

When a dielectric material is subjected to an alternating electric field by applying AC, the electric polarisation changes direction continuously as the dipole orientation changes direction.

1. For low frequencies (in the audio frequency range), the change over is quick and responds to variations in electric fields. Here all the polarisabilities are present as shown in the diagram. $P = P_e + P_i + P_o$. In the diagram k' is the dielectric constant and k'' is the absorption loss.
2. As the frequency is increased (in the radio and microwave region), the orientational (dipolar) polarisation does not take place and net polarisation decreases. It is given by $P = P_e + P_i$. This results in decrease of dielectric constant ϵ_r .



3. In the infrared region ionic polarisation ceases. In this region and the optical frequencies only electronic polarisation contributes to the dielectric constant. This contribution decreases further with increase in frequency.

The absorption peaks occur at specific frequencies when the frequency of the applied field is equal to the relaxation frequency of the dipoles, maximum absorption from the electric field occurs. (The average time taken by the dipoles to orient in the direction of field due to its reversal is the relaxation time and its reciprocal is the relaxation frequency).

Dielectric breakdown

If a dielectric material is placed in a very strong electric field, electrons can be removed from their corresponding atoms causing large currents to flow and damaging the material. This phenomenon is called **dielectric breakdown**.

This results in the material behaving as conductor instead of insulator.

The value of the electric field at which *dielectric breakdown* occurs is called the *dielectric strength* of the material. Dielectric strength is the maximum electric field that the dielectric material can withstand without suffering electrical breakdown. The *dielectric strength* of a material is denoted by the symbol E_{BR} . It is the breakdown voltage per unit thickness of the material.

The dielectric strength of a material may vary by several orders of magnitude depending on various factors including the exact composition of the material. Usually dielectric breakdown does not permanently damage gaseous or liquid dielectrics, but does ruin solid dielectrics.

The different types of dielectric breakdown are

1. **Intrinsic breakdown** : This occurs due to breaking of covalent bonds and electrons moving from valance to conduction band resulting in conduction.
2. **Thermal breakdown** : Due to high temperature, the bonds break and random motion of electrons increase. If the heat supplied is greater than the heat dissipated the remaining heat will lead to electrons moving out of atoms resulting in large current.
3. **Discharge breakdown** : This occurs due to presence of gas bubbles in the solid medium forming ions resulting in current.
4. **Electrochemical breakdown** : This is also due to heat and chemical reactions.
5. **Defect breakdown** : This is due to defects such as cracks, dislocations, discontinuities and presence of impurities.

Air has dielectric strength of $3 \times 10^6 \text{ Vm}^{-1}$ and paper has $16 \times 10^6 \text{ Vm}^{-1}$.

Ferroelectricity

It is a property of certain materials in which they possess spontaneous electric polarisation even in the absence of electric field and can be reversed by the application of external electric field.

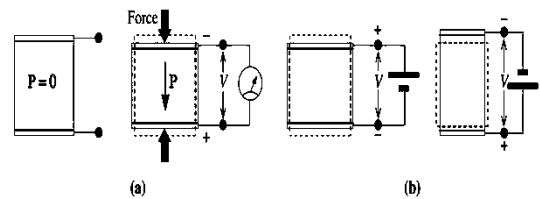
The materials are referred as ferroelectrics which are analogous in electrical properties to magnetic properties of ferromagnetics. Eg. Barium titanate, potassium dihydro phosphate ...

Ferroelectrics exhibit electric hysteresis similar to magnetic hysteresis and obey Curie – Weiss law.

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Piezoelectricity

It is the electric charge that accumulates in certain solid materials (such as crystals, certain ceramics, and biological matter such as bone, DNA and various proteins) in response to applied mechanical stress.



The word *piezoelectricity* means electricity resulting from pressure and latent heat.

A material that develops voltage due to application of a stress and develops strain when an electric field is applied is called Piezoelectricity. Diagram (a) indicates the development of voltage due to stress and (b) indicates the strain or deformation due to voltage.

The process of creating electric polarization by mechanical stress is called as piezo electric effect.

This process is used in conversion of mechanical energy into electrical energy and also electrical energy into mechanical energy. According to inverse piezo electric effect, when an electric stress is applied, the material becomes strained. This strain is directly proportional to the applied field. Examples: quartz crystal , Rochelle salt etc.,

Applications :

1. Piezo electric materials or peizo electric semiconductors such as Gas, Zno and CdS are finding applications in ultrasonic amplifiers.
2. Piezoelectric materials are used in quartz watches.
3. They are also used as oscillators. They are used as transducers to measure pressure very accurately.
4. Piezoelectricity is used in the production and detection of sound, piezoelectric inkjet printing, generation of high voltages, electronic frequency generation, It forms the basis for a number of scientific instrumental techniques with atomic resolution, the scanning probe microscopies, such as STM, AFM, etc...
5. It also finds everyday uses such as acting as the ignition source for cigarette lighters, push-start propane barbecues etc... .

Electrostriction

When any material undergoes polarization (due to an applied electric field), its ions and electronic clouds are displaced, causing the development of a mechanical strain in the material.

This phenomenon is known as the electrostriction. The pressure experienced by the dielectric is directly proportional to the square of the applied electric field and is not altered when the direction of the electric field is reversed.

Electret

Electrets are materials that have a quasi-permanent electric field at their surfaces due to either an imbalance of charge at the surface (space charge electrets) or to aligned dipoles through the bulk of the material (dipolar electrets).

The term *quasi-permanent* implies that the amount of charges stored in the material does not remain the same permanently, but decays very slowly depending on the

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situation, and the decay time is normally much longer than the time period over which the electret is in use.

The electrostatic properties of electrets have been exploited for many applications including xerography, powder coating, and electrostatic precipitation.

Electret is the material that retains its electric polarization after being subjected to a strong electric field. The positive charge within the material becomes permanently displaced in the direction of the field, and the negative charge becomes permanently displaced in the direction opposite to the field. One end of the electret is somewhat positive, and the other is somewhat negative, though the net charge remains zero.

Electrets are prepared from certain waxes, plastics, and ceramics, the individual molecules of which are permanently polarized but are randomly arranged before being subjected to an electric field so that there is no overall polarization in the material.

The strong electric field (approximately 1,000,000 volts per metre) rotates the polar molecules into an alignment that persists when the external field is removed. Sometimes electrets are made by allowing a molten material to solidify in a strong electric field.

Example for electrets is a microphone. An **electret microphone** is a type of electrostatic capacitor-based microphone, which eliminates the need for a polarizing power supply by using a permanently charged material.

An electret carries a permanent electric field just as a permanent magnet carries a permanent magnetic field. One way to think of an electret is as a capacitor that always carries a charge and that you never have to recharge.

PART A Descriptive questions

- (a) What are dielectrics? Distinguish between polar and nonpolar dielectrics.
(b) What is dielectric constant? Arrive at the expression for the static dielectric constant.
- What is electrical polarisation and electrical susceptibility? Arrive at the expression for dielectric polarisation.
- (a) Explain in brief the different types of dielectric polarisation.
(b) Derive an expression for the electronic polarisability.
- What is internal field? Arrive at the expression for the internal field of a dielectric.
- (a) Derive Clausius Mossotti equation for a 3-dimensional cubic lattice.
(b) Explain how the dielectric constant varies with frequency.
- What is dielectric breakdown? Explain the different types of dielectric breakdown.
- (a) What is ferroelectricity? Explain the terms- Electrostriction and electret.
(b) What is piezoelectricity? Mention applications of piezoelectric materials.

Numerical problems

- A parallel plate capacitor is of area 0.04 m^2 with plate separation of 2 mm . A voltage of 10 V is applied across the capacitor. Find the capacitance of the capacitor. If a dielectric medium of relative permittivity 6 is introduced between the plates, find the capacitance, electric displacement and dielectric polarisation. ($\epsilon_0 = 8.854 \times 10^{-12} \text{ Fm}^{-1}$)

$$\left[\begin{array}{l} C_0 = \frac{\epsilon_0 A}{d} = 0.177 \times 10^{-9} \text{ F}, \quad C = \epsilon_r C_0 = 1.06 \times 10^{-9} \text{ F}, \quad E = \frac{V}{d} = 5 \times 10^3 \text{ Vm}^{-1}, \quad D = \\ \epsilon_0 \epsilon_r E = 0.265 \times 10^{-6} \text{ Cm}^{-2} \quad P = D - \epsilon_0 E = 0.212 \times 10^{-6} \text{ Cm}^{-2} \end{array} \right]$$

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2. The dielectric constant of a gas at NTP is 1.000456. Find the electronic polarisability. If the gas contains 2.7×10^{23} atoms per cubic metre, find the radius of the atom.
 [$P = \epsilon_0 E(\epsilon_r - 1)$, $P_e = N\alpha_e E$ comparing $N\alpha_e E = \epsilon_0 E(\epsilon_r - 1)$ or $\alpha_e = \frac{\epsilon_0(\epsilon_r - 1)}{N} = 1.49 \times 10^{-38} \text{ Fm}^2$, $\alpha_e = 4\pi\epsilon_0 R^3$ $R = 0.049 \times 10^{-9} \text{ m}$]
3. Dielectric constant of sulphur is 3.4. Find electronic polarisability if the volume density of sulphur is $2 \times 10^3 \text{ kgm}^{-3}$ and atomic weight is 32.
 [$\frac{(\epsilon_r - 1)}{(\epsilon_r + 2)} = \frac{N\alpha_e}{3\epsilon_0}$, $\alpha_e = \frac{3\epsilon_0 (\epsilon_r - 1)}{N (\epsilon_r + 2)}$ where $N = \frac{\text{density } (\rho) \times \text{Avogadro No. } (N_A)}{\text{atomic weight}} = \frac{2000 \times 6.023 \times 10^{23}}{32 \times 10^{-3}}$
 $\alpha_e = 8 \times 10^{-40} \text{ Fm}^2$]
4. The electronic polarisability of an atom is $7 \times 10^{-40} \text{ F m}^{-2}$. Find dielectric constant if the number of atoms per cubic metre is 3×10^{28} .
 [$N\alpha_e E = \epsilon_0 E(\epsilon_r - 1)$, $\epsilon_r - 1 = \frac{N\alpha_e}{\epsilon_0}$ or $\epsilon_r = \frac{N\alpha_e}{\epsilon_0} + 1 = 3.37$]
5. The radius of helium atom is about 0.55 \AA . Calculate the polarisability of helium and its relative permittivity. The number of helium atoms in a volume of 1 m^3 is 2.7×10^{28} .
 [$\alpha_e = 4\pi\epsilon_0 R^3 = 18.5 \times 10^{-42} \text{ Fm}^2$, $\epsilon_r = \frac{N\alpha_e}{\epsilon_0} + 1 = 1.056$]
6. Calculate the induced dipole moment per unit volume (also called electric polarisation) of helium gas when it is placed in an electric field of $6 \times 10^5 \text{ Vm}^{-1}$. The atomic polarisability of helium is $0.18 \times 10^{-40} \text{ Fm}^2$ and the concentration of helium atom is $2.6 \times 10^{25} / \text{m}^3$.
 [$\mu_i = P = N\alpha_e E = 2.6 \times 10^{25} \times 0.18 \times 10^{-40} \times 6 \times 10^5 = 2.81 \times 10^{-10} \text{ Cm}^{-2}$]
7. The number of atoms in a volume of 1 m^3 is 3×10^{28} . The electronic polarisability is 10^{-40} Fm^2 . Find the relative permittivity of the given system. [$\epsilon_r = \frac{N\alpha_e}{\epsilon_0} + 1 = 1.338$]
8. Calculate the capacitance of Al_2O_3 that is $0.5 \text{ }\mu\text{m}$ thick and 2000 mm^2 area and having dielectric constant of 1.0034. [$C = \frac{\epsilon_0 \epsilon_r A}{d} = 3.54 \times 10^{-8} \text{ F}$]
9. If an ionic crystal is subjected to a electric field of 1000 Vm^{-1} produces a polarization of $4.3 \times 10^{-8} \text{ Cm}^{-2}$, calculate the dielectric constant. [$P = \epsilon_0 E(\epsilon_r - 1)$ or $\epsilon_r = 1 + \frac{P}{\epsilon_0 E} = 5.856$]
10. Calculate the polarization produced in dielectric medium of dielectric constant 6 when it is subjected to an electric field of 100 Vm^{-1} . [$P = \epsilon_0 E(\epsilon_r - 1) = 4.427 \times 10^{-9} \text{ Cm}^{-2}$]
11. Calculate the polarization and susceptibility in a crystal when 3V is applied across a dielectric of 1 mm thickness. Given K of crystal is 4.
 [$P = \epsilon_0 E(\epsilon_r - 1)$ or $P = \epsilon_0 \frac{V}{d} (\epsilon_r - 1) = 8.854 \times 10^{-12} \times \frac{3}{1 \times 10^{-3}} \times (4 - 1) = 79.6 \times 10^{-9} \text{ Cm}^{-2}$
 $\chi = \epsilon_r - 1 = 3$ (Here dielectric constant $K = \epsilon_r$)]
12. A parallel plate capacitor is made up of a dielectric of thickness 2 mm and dielectric constant 8. If the potential difference across the capacitor is 50 V, calculate the polarization and electric displacement.
 [$P = \epsilon_0 E(\epsilon_r - 1)$ or $P = \epsilon_0 \frac{V}{d} (\epsilon_r - 1) = 1.55 \times 10^{-6} \text{ Cm}^{-2}$, $D = \epsilon E = \epsilon_0 \epsilon_r E = \epsilon_0 \epsilon_r \times \frac{V}{d}$
 $D = 1.7710 \times 10^{-6} \text{ Cm}^{-2}$]