Classical Theory of Paramagnetism or Langevin's Theory

Consider a paramagnetic gas containing N atoms/vol each associated with a permanent magnetic moment μ.

In an applied magnetic field, a net magnetisation results due to the allignment of the magnetic moments. The energy of interaction of the moment μ with the applied magnetic field B is given by

$$E = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\theta \quad \text{--- (1)}$$

θ = angle between μ & B.

Energy is least, when μ & B are parallel ie magnetic field tends to allign the magnetic dipoles in its own direction, but at ordinary temperatures, particles are subjected to thermal agitation, therefore they are ~~the~~ randomly oriented.

The probability that a magnetic dipole is inclined at an angle θ to the field direction in thermal equilibrium is given by M.B distribution function as

$$f(E) = A\exp\left(\frac{-E}{K_B T}\right)$$

②

$$f(E) = A \exp\left(\frac{\mu B\cos\theta}{K_B T}\right) \quad -②$$

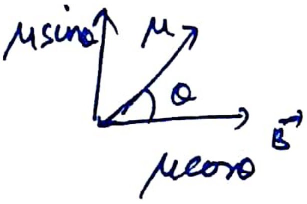
A = Normalisation constant

The total number of magnetic dipoles n, inclined at angle θ to the field B are given as

$$n = n_0 \exp\left[\frac{\mu B\cos\theta}{K_B T}\right] \quad -③ \quad [\because n \propto \text{Probability of distribution}]$$

The average magnetic moment ⟨μ⟩ in the field direction will be obtained by dividing the sum of the resolved components of the magnetic moments of all the dipoles by the total number of dipoles.

$$\langle \mu \rangle = \frac{\int \mu \cos\theta \, dn}{\int dn} \quad -④$$



→ Note that component μcosθ is in the direction of magnetic field B

$$\langle \mu \rangle = \frac{\int_0^\pi \mu_m \cos\theta . \underbrace{\exp\left(\frac{\mu B\cos\theta}{K_B T}\right) \sin\theta \, d\theta}_{dn \{\text{from eq ③}\}}}{\int_0^\pi \exp\left(\frac{\mu B\cos\theta}{K_B T}\right) \sin\theta \, d\theta}$$

Putting $x = \frac{\mu B}{k_B T}$ $\quad s = \cos\theta$

$ds = -\sin\theta \, d\theta$

$\theta = 0 \quad s \to 1$

$\theta = \pi \quad s \to -1$

$$\langle \mu \rangle = \frac{\int_{-1}^{+1} \mu e^{sx} \cdot s \, ds}{\int_{-1}^{+1} e^{sx} \, ds}$$

Evaluating the integral, we get

$$\langle \mu \rangle = \mu \left[\frac{e^{x} + e^{-x}}{e^{x} - e^{-x}} - \frac{1}{x} \right]$$

$$= \mu \left[\coth x - \frac{1}{x} \right] = \mu L(x)$$

$\langle \mu \rangle = \mu L(x)$, where $L(x)$ is called Langevin's function.

If N = no. of atoms/vol.

$\therefore$ Magnetisation i.e the total magnetic moment per unit volume of the substance is given by

$$M = N \langle \mu \rangle = N \mu L(x) \quad - (5)$$

$L(x)$ is evaluated for the condition

$x << 1$ or $\frac{\mu B}{K_B T} << 1$

ii at high temperature and small magnetic field

$$L(x)\big|_{x\to 0} = \left|\coth x - \frac{1}{x}\right|_{x\to 0} = \left|\frac{e^x + e^{-x}}{e^x - e^{-x}} - \frac{1}{x}\right|_{x\to 0}$$

$$L(x)\big|_{x\to 0} \cong \frac{x}{3} = \frac{\mu B}{3K_BT}$$

$$\therefore \quad M \cong \frac{N\mu^2 B}{3K_BT} \qquad \vec{B} = \mu_0\vec{H}$$

$$\boxed{M = \frac{\mu_0 N\mu^2 H}{3K_BT}} \quad \text{—(6)}$$

The Paramagnetic susceptibility

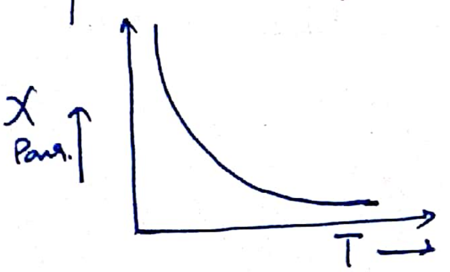
$$\chi = \frac{M}{H} = \frac{\mu_0 N\mu^2}{3K_BT}$$

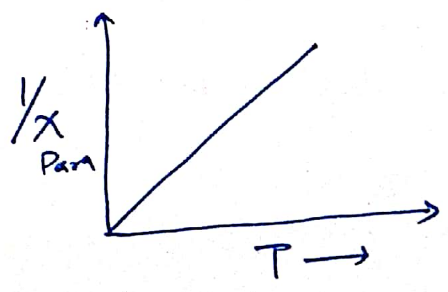
$$\therefore \quad \boxed{\chi_{Para} = \frac{C}{T}} \quad \text{—(7)} \quad \underline{\text{Curie's law}}$$

where $C = \dfrac{\mu_0 N\mu^2}{3K_B}$ is called Curie's Constt.

& eq (7) is called Curie's law

∴ Curie's law is applied in the limit $\mu B <<< K_BT$ and it holds good at high temperatures





Now for large values of x ie

if $x >> 1$, ie $\frac{\mu B}{k_B T} >> 1$

ie at low temperature & high magnetic field

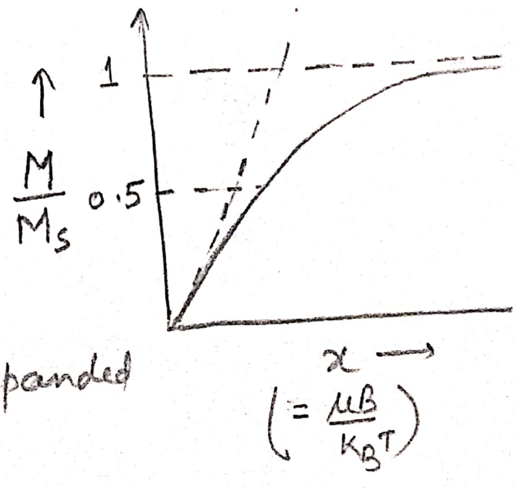
$L(x) \rightarrow 1$ (approaches unity)

then eq 5 becomes

$$\boxed{M = N\mu = M_s} = \text{Saturation Magnetisation} \quad (8)$$

This is the saturation condition which corresponds to the complete allignment of the magnetic dipoles in the field direction.

In practice, the approach to the saturation of magnetisation is never attained in gases, for practical purpose, the initial part of the curve is considered.



NOTE:

For small x, graph is linear and coincides as tangent to the curve. $L(x) = \coth x - \frac{1}{x}$

for small x, $\coth x$ is expanded in power series of x

$$= \left(\cancel{\frac{1}{x}} + \frac{x}{3}\right) - \cancel{\frac{1}{x}} = \frac{x}{3}$$

$\therefore L(x) \simeq \frac{x}{3}$, which we have substituted in equation 5.